

Properties of Real Numbers

Commutative Property of Addition

$$a + b = b + a$$

$$2 + 3 = 3 + 2$$

Identity Property of Addition

$$a + 0 = a$$

$$3 + 0 = 3$$

Commutative Property of Multiplication

$$a \cdot b = b \cdot a$$

$$2 \cdot 3 = 3 \cdot 2$$

Multiplicative Property of 0

$$a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$$

$$5 \cdot 0 = 0 \text{ and } 0 \cdot 5 = 0$$

Multiplicative Property of -1

$$a \cdot (-1) = -1 \cdot a = -a$$

$$4 \cdot (-1) = -1 \cdot 4 = -4$$

Distributive Property of Multiplication over Addition

$$a(b + c) = ab + ac$$

$$2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$$

Reflexive Property

$$a = a$$

$$8 = 8$$

Transitive Property

$$\text{If } a = b \text{ and } b = c, \text{ then } a = c$$

$$\text{If } a = b \text{ and } b = 2, \text{ then } a = 2$$

Addition Property of Equality

$$\text{If } a = b, \text{ then } a + c = b + c$$

Multiplication Property of Equality

$$\text{If } a = b, \text{ and } c \neq 0, \text{ then } a \cdot c = b \cdot c$$

Associative Property of Addition

$$a + (b + c) = (a + b) + c$$

$$4 + (3 + 5) = (4 + 3) + 5$$

Additive Property of Opposites

$$a + (-a) = 0$$

$$4 + (-4) = 0$$

Associative Property of Multiplication

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

$$2 \cdot (4 \cdot 6) = (2 \cdot 4) \cdot 6$$

Identity Property of Multiplication

$$a \cdot 1 = a \text{ and } 1 \cdot a = a$$

$$7 \cdot 1 = 7 \text{ and } 1 \cdot 7 = 7$$

Property of Reciprocals

$$\frac{a}{b} \cdot \frac{b}{a} = 1$$

$$\frac{2}{3} \cdot \frac{3}{2} = 1$$

Distributive Property of Multiplication over Subtraction

$$a(b - c) = ab - ac$$

$$3(7 - 4) = 3 \cdot 7 - 3 \cdot 4$$

Symmetric Property

$$\text{If } a = b, \text{ then } b = a$$

$$\text{If } a = 3, \text{ then } 3 = a$$

Subtraction Property of Equality

$$\text{If } a = b, \text{ then } a - c = b - c$$

Division Property of Equality

$$\text{If } a = b, \text{ and } c \neq 0, \text{ then } a \div c = b \div c$$

Properties of Exponents

Product of Powers

$$a^m \cdot a^n = a^{m+n}$$

$$2^3 \cdot 2^4 = 2^{3+4}$$

Power of a Product

$$(a \cdot b)^m = a^m \cdot b^m$$

$$(2 \cdot 3)^4 = 2^4 \cdot 3^4$$

Power of a Power

$$(a^m)^n = a^{m \cdot n}$$

$$(3^2)^4 = 3^{2 \cdot 4}$$

Power of a Quotient

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, \quad b \neq 0$$

Zero Exponent

$$a^0 = 1$$

$$5^0 = 1$$

Quotient of Powers

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a^5}{a^3} = a^{5-3}$$

Negative Exponent

$$a^{-n} = \frac{1}{a^n}, \quad a \neq 0$$

$$2^{-3} = \frac{1}{2^3}$$

Properties of Logarithms

Definition of Logarithm Base b

$$\log_b y = x \text{ if and only if } b^x = y$$

Common Logarithm

$$\log_{10} x = \log x$$

Natural Logarithm

$$\log_e x = \ln x$$

Product Property

$$\log_b mn = \log_b m + \log_b n$$

where $b, m,$ and $n > 0$ and $b \neq 1$

$$\log_2 (3 \cdot 7) = \log_2 3 + \log_2 7$$

Power Property

$$\log_b m^n = n \cdot \log_b m$$

$$\log_3 2^4 = 4 \cdot \log_3 2$$

One-to-One Property

If $\log_b x = \log_b y$, then $x = y$,

where $b, x,$ and $y > 0$ and $b \neq 1$

$$\log_3 (2x + 3) = \log_3 (x + 5)$$

$$2x + 3 = x + 5$$

$$x = 2$$

Special Logarithmic Values

$$\log_b 1 = 0 \text{ because } b^0 = 1$$

$$\log_b b = 1 \text{ because } b^1 = b$$

$$\ln 1 = 0 \text{ because } e^0 = 1$$

$$\ln e = 1 \text{ because } e^1 = e$$

Quotient Property

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

where $b, m,$ and $n > 0$ and $b \neq 1$

$$\log_5 \frac{3}{7} = \log_5 3 - \log_5 7$$

Inverse Properties

$$\log_b b^x = x \qquad b^{\log_b x} = x$$

$$\log_3 3^2 = 2 \qquad 5^{\log_5 3} = 3$$

Change of Base

$$\log_a x = \frac{\log x}{\log a}$$

$$\log_2 5 = \frac{\log 5}{\log 2} \approx \frac{0.699}{0.301} \approx 2.322$$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\log_3 7 = \frac{\ln 7}{\ln 3} \approx \frac{1.946}{1.099} \approx 1.771$$