

Equivalent Fractions

To find equivalent fractions multiply or divide both the numerator and the denominator by the same number.

Examples:

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

To write a fraction in simplest form, write an equivalent fraction by dividing both the numerator and the denominator by the greatest common factor.

Example:

Write the fraction $\frac{6}{9}$ in simplest form.

The greatest common factor of 6 and 9 is 3. Divide both the numerator and the denominator by 3.

$$\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$

Mixed Numbers

To write a mixed number as an improper fraction, multiply the denominator of the fraction by the whole number and add the numerator to the product. Write the result over the denominator.

Example:

$$3\frac{2}{5} = \frac{3 \times 5 + 2}{5} = \frac{17}{5}$$

To write an improper fraction as a mixed number, divide the numerator by the denominator. The quotient is the whole portion of the mixed number. The remainder is the numerator and the divisor is the denominator of the fraction portion.

Example:

$$\frac{14}{5} = 2\frac{4}{5} \quad 14 \div 5 = 2 \text{ R}4$$

Reciprocals

The product of a fraction and its reciprocal is 1. To write the reciprocal of a fraction switch the position of the numbers in the numerator and the denominator.

Example:

The reciprocal of the fraction $\frac{2}{3}$ is $\frac{3}{2}$ because $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.

Reciprocals (continued)

Write a mixed number as an improper fraction to find its reciprocal.

Example:

The reciprocal of the mixed number $2\frac{3}{4}$ is $\frac{4}{11}$ because $2\frac{3}{4} = \frac{11}{4}$ and

$$\frac{11}{4} \times \frac{4}{11} = \frac{44}{44} = 1.$$

Converting a Fraction to a Decimal

To convert a fraction to a decimal, divide the numerator by the denominator.

Examples:

The decimal equivalent of the fraction $\frac{3}{8}$ is 0.375 because $8 \overline{)3.000} \begin{array}{r} 0.375 \\ \end{array}$.

The decimal equivalent of the mixed number $2\frac{3}{4}$ is 2.75 because $4 \overline{)3.00} \begin{array}{r} 0.75 \\ \end{array}$.

Converting a Decimal to a Fraction

To convert a decimal number to a fraction, write the digits to the right of the decimal point as the numerator of the fraction. The denominator of the fraction is the place value of the last digit. Write the fraction in simplest form.

Example:

The fractional equivalent of the decimal number 0.125 is $\frac{1}{8}$ because the digits to the right of the decimal point are 125 and the 5 is in the thousandths place.

$$\frac{125}{1000} = \frac{1}{8}$$

To convert a decimal number greater than 1 to a mixed number, write the digits to the left of the decimal point as the whole number portion of the mixed number. Convert the digits to the right of the decimal point to a fraction.

Example:

The fractional equivalent of the decimal number 3.42 is $3\frac{21}{50}$ because the digits to the right of the decimal point are 42 and the 2 is in the hundredths place.

$$3.42 = 3\frac{42}{100} = 3\frac{21}{50}$$

Adding Fractions

Like Denominators

Step 1: Add the numerators and write the sum over the denominator.

Step 2: Write the answer in simplest form.

Example:

Add: $\frac{3}{8} + \frac{1}{8}$

Step 1: Add the numerators and write the sum over the denominator.

$$\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$$

Step 2: Write the answer in simplest form.

$$\frac{4}{8} = \frac{1}{2}$$

Unlike Denominators

Step 1: Find the least common denominator (LCD). The LCD is the smallest multiple that the denominators have in common.

Step 2: Write equivalent fractions using the LCD.

Step 3: Add the numerators and write the sum over the denominator.

Step 4: Write the answer in simplest form.

Example:

Add: $\frac{1}{3} + \frac{3}{4}$

Step 1: Find the LCD.

The smallest multiple that 3 and 4 have in common is 12.

Step 2: Write equivalent fractions using a denominator of 12.

$$\frac{1}{3} = \frac{4}{12} \quad \frac{3}{4} = \frac{9}{12}$$

Step 3: Add the numerators and write the sum over the denominator.

$$\frac{4}{12} + \frac{9}{12} = \frac{13}{12}$$

Step 4: Write the answer in simplest form.

$$\frac{13}{12} = 1\frac{1}{12}$$

Addition Problem

Write the problem here:

Step 1: Find the least common denominator (LCD)

List multiples of the 1st denominator here:

List multiples of the 2nd denominator here:

The LCD is:

Step 2: Write equivalent fractions using the LCD

1st fraction

2nd Fraction

Step 3: Find the sum of the numerators

Step 4: Express the answer in simplest form

Subtracting Fractions

Like Denominators

Step 1: Subtract the numerators and write the difference over the denominator.

Step 2: Write the answer in simplest form.

Example:

$$\text{Subtract: } \frac{5}{8} - \frac{3}{8}$$

Step 1: Subtract the numerators and write the difference over the denominator.

$$\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$$

Step 2: Write the answer in simplest form.

$$\frac{2}{8} = \frac{1}{4}$$

Unlike Denominators

Step 1: Find the least common denominator (LCD). The LCD is the smallest multiple that the denominators have in common.

Step 2: Write equivalent fractions using the LCD.

Step 3: Subtract the numerators and write the difference over the denominator.

Step 4: Write the answer in simplest form.

Example:

$$\text{Subtract: } \frac{4}{5} - \frac{2}{3}$$

Step 1: Find the LCD.

The smallest multiple that 5 and 3 have in common is 15.

Step 2: Write equivalent fractions using a denominator of 15

$$\frac{4}{5} = \frac{12}{15} \quad \frac{2}{3} = \frac{10}{15}$$

Step 3: Subtract the numerators and write the difference over the denominator.

$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Step 4: Write the answer in simplest form.

The answer $\frac{2}{15}$ is in simplest form.

Subtraction Problem

Write the problem here:

Step 1: Find the least common denominator (LCD)

List multiples of the 1st denominator here:

List multiples of the 2nd denominator here:

The LCD is:

Step 2: Write equivalent fractions using the LCD

1st fraction

2nd Fraction

Step 3: Find the difference of the numerators

Step 4: Express the answer in simplest form

Multiplying Fractions

Multiply a fraction by a fraction

Step 1: Multiply the numerators.

Step 2: Multiply the denominators.

Step 3: Write the answer in simplest form.

Example:

Multiply: $\frac{3}{4} \times \frac{2}{3}$

Step 1: Multiply the numerators.

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Step 2: Multiply the denominators.

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Step 3: Write the answer in simplest form.

$$\frac{6}{12} = \frac{1}{2}$$

Multiply mixed numbers

Step 1: Write the mixed numbers as improper fractions.

Step 2: Multiply the numerators.

Step 3: Multiply the denominators.

Step 4: Write the answer in simplest form.

Example:

Multiply: $3\frac{1}{3} \times 2\frac{1}{2}$

Step 1: Write the mixed numbers as improper fractions.

$$3\frac{1}{3} = \frac{10}{3} \qquad 2\frac{1}{2} = \frac{5}{2}$$

Step 2: Multiply the numerators.

$$\frac{10}{3} \times \frac{5}{2} = \frac{50}{6}$$

Step 3: Multiply the denominators.

$$\frac{10}{3} \times \frac{5}{2} = \frac{50}{6}$$

Step 4: Write the answer in simplest form.

$$\frac{50}{6} = 8\frac{2}{6} = 8\frac{1}{3}$$

Multiplication Problem

Write the problem here:

Step 1: Multiply the numerators

The product of the numerators is:

Step 2: Multiply the denominators

The product of the denominators is:

Step 3: Write a fraction using the products

Step 4: Express the answer in simplest form

Dividing Fractions

Divide a fraction by a fraction

Step 1: Write the division expression as a multiplication expression using the reciprocal of the divisor.

Step 2: Multiply the numerators and the denominators.

Step 3: Write the answer in simplest form.

Example:

$$\text{Divide: } \frac{5}{8} \div \frac{3}{4}$$

Step 1: Write the division expression as a multiplication expression using the reciprocal of the divisor.

$$\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3}$$

Step 2: Multiply the numerators and the denominators.

$$\frac{5}{8} \times \frac{4}{3} = \frac{20}{24}$$

Step 3: Write the answer in simplest form.

$$\frac{20}{24} = \frac{5}{6}$$

Divide mixed numbers

Step 1: Write the mixed numbers as improper fractions.

Step 2: Write the division expression as a multiplication expression using the reciprocal of the divisor.

Step 3: Multiply the numerators and the denominators.

Step 4: Write the answer in simplest form.

Example:

$$\text{Divide: } 2\frac{2}{3} \div 1\frac{1}{5}$$

Step 1: Write the mixed numbers as improper fractions.

$$2\frac{2}{3} = \frac{8}{3} \quad 1\frac{1}{5} = \frac{6}{5}$$

Step 2: Write the division expression as a multiplication expression using the reciprocal of the divisor.

$$\frac{8}{3} \div \frac{6}{5} = \frac{8}{3} \times \frac{5}{6}$$

Step 3: Multiply the numerators and the denominators.

$$\frac{8}{3} \times \frac{5}{6} = \frac{40}{18}$$

Step 4: Write the answer in simplest form.

$$\frac{40}{18} = 2\frac{4}{18} = 2\frac{2}{9}$$

Division Problem

Write the problem here:

Find the reciprocal of the divisor

Write the divisor here:

Write the reciprocal of the divisor here:

Step 1: Write the division problem as a multiplication problem using the reciprocal

Step 2: Find the product

Write the product of the numerators:

Write the product of the denominators:

Write a fraction using the products

Step 3: Express the answer in simplest form

Equivalent Algebraic Fractions

To find equivalent algebraic fractions multiply or divide the numerator and the denominator by the same term.

Examples:

$$\frac{3x}{5y} = \frac{3x \cdot 2x}{5y \cdot 2x} = \frac{6x^2}{10xy} \qquad \frac{9x^2y^2}{6xy^3} = \frac{9x^2y^2 \div 3xy^2}{6xy^3 \div 3xy^2} = \frac{3x}{2y}$$

To write an algebraic fraction in simplest form, write an equivalent fraction by dividing the numerator and the denominator by the greatest common factor.

Example:

Write the algebraic fraction $\frac{8m^2n^2}{6mn^3}$ in simplest form.

The greatest common factor of $8m^2n^2$ and $6mn^3$ is $2mn^2$. Divide both the numerator and the denominator by $2mn^2$.

$$\frac{8m^2n^2}{6mn^3} = \frac{8m^2n^2 \div 2mn^2}{6mn^3 \div 2mn^2} = \frac{4m}{3n}$$

Least Common Denominator (LCD) – Algebraic Fractions

The least common denominator (LCD) is the smallest multiple that the denominators have in common. Factor each denominator and find the least common multiple.

Example:

Find the LCD of $\frac{3}{6x^2}$ and $\frac{x}{4x^2 - 12x}$.

Factor the denominators.

$$6x^2 = 2 \cdot 3 \cdot x \cdot x$$

$$4x^2 - 12x = 4x(x - 3) = 2 \cdot 2 \cdot x \cdot (x - 3)$$

The smallest multiple that is common to both denominators is

$$2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot (x - 3) = 12x^2(x - 3)$$

The LCD is $12x^2(x - 3)$.

Adding and Subtracting Algebraic Fractions

Like Denominators

Step 1: Add or subtract the numerators and write the result over the denominator.

Step 2: Write the answer in simplest form.

Example:

Add: $\frac{3}{4x} + \frac{5}{4x}$

Step 1: Add the numerators and write the sum over the denominator.

$$\frac{3}{4x} + \frac{5}{4x} = \frac{8}{4x}$$

Step 2: Write the answer in simplest form.

$$\frac{8}{4x} = \frac{2}{x}$$

Unlike Denominators

Step 1: Factor the denominators and find the least common denominator (LCD).

Step 2: Write equivalent fractions using the LCD.

Step 3: Add or subtract the numerators and write the result over the denominator.

Step 4: Write the answer in simplest form.

Example:

Add: $\frac{3}{4x^2} + \frac{x}{3x^2 - 12x}$

Step 1: Find the LCD.

Factor the second denominator: $3x^2 - 12x = 3x(x - 4)$. The smallest multiple that $4x^2$ and $3x(x - 4)$ have in common is $12x^2(x - 4)$.

Step 2: Write equivalent fractions using a denominator of $12x^2(x - 4)$.

$$\frac{3}{4x^2} \cdot \frac{3(x-4)}{3(x-4)} = \frac{9(x-4)}{12x^2(x-4)} = \frac{9x-36}{12x^2(x-4)}$$

$$\frac{x}{3x(x-4)} \cdot \frac{4x}{4x} = \frac{4x^2}{12x^2(x-4)}$$

Step 3: Add the numerators and write the sum over the denominator.

$$\frac{9x-36}{12x^2(x-4)} + \frac{4x^2}{12x^2(x-4)} = \frac{9x-36+4x^2}{12x^2(x-4)}$$

Step 4: Write the answer in simplest form.

$$\frac{4x^2 + 9x - 36}{12x^3 - 48x^2}$$

Multiplying Algebraic Fractions

Step 1: Multiply the numerators.

Step 2: Multiply the denominators.

Step 3: Write the answer in simplest form.

Example:

$$\text{Multiply: } \frac{3a^2}{2b} \cdot \frac{4}{ab}$$

Step 1: Multiply the numerators.

$$\frac{3a^2}{2b} \cdot \frac{4}{ab} = \frac{12a^2}{2ab^2}$$

Step 2: Multiply the denominators.

$$\frac{3a^2}{2b} \cdot \frac{4}{ab} = \frac{12a^2}{2ab^2}$$

Step 3: Write the product in simplest form.

$$\frac{12a^2}{2ab^2} = \frac{6a}{b^2}$$

Dividing Algebraic Fractions

Step 1: Write the division expression as a multiplication expression using the reciprocal of the divisor.

Step 2: Multiply the numerators and the denominators.

Step 3: Write the answer in simplest form.

Example:

$$\text{Divide: } \frac{5y^2}{x+2} \div \frac{y}{2}$$

Step 1: Write the division expression as a multiplication expression using the reciprocal of the divisor.

$$\frac{5y^2}{x+2} \div \frac{y}{2} = \frac{5y^2}{x+2} \cdot \frac{2}{y}$$

Step 2: Multiply the numerators and the denominators.

$$\frac{5y^2}{x+2} \cdot \frac{2}{y} = \frac{10y^2}{y(x+2)}$$

Step 3: Write the answer in simplest form.

$$\frac{10y^2}{y(x+2)} = \frac{10y}{x+2}$$

Simplifying Complex Fractions

Step 1: Simplify the numerator.

Step 2: Simplify the denominator.

Step 3: Write the simplified expression as a multiplication expression using the reciprocal of the divisor (the denominator of the complex fraction).

Step 4: Write the answer in simplest form.

Example:

$$\text{Simplify: } \frac{\frac{3}{5} + 2\frac{1}{2}}{\frac{2}{3} - \frac{1}{2}}$$

Step 1: Simplify the numerator.

$$\frac{3}{5} + 2\frac{1}{2} = \frac{3}{5} + \frac{5}{2} = \frac{6}{10} + \frac{25}{10} = \frac{31}{10}$$

Step 2: Simplify the denominator.

$$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

Step 3: Write the simplified expression as a multiplication expression using the reciprocal of the divisor.

$$\frac{\frac{31}{10}}{\frac{1}{6}} = \frac{31}{10} \cdot \frac{6}{1} = \frac{186}{10}$$

Step 4: Write the answer in simplest form.

$$\frac{186}{10} = 18\frac{6}{10} = 18\frac{3}{5}$$