

## Strategies for Addition You Might Encounter

### Use a Number Chart

Start with the first addend. Decompose the second addend into tens and ones. Move down the chart to add tens and to the right to add ones.

$$23 + 46 = 69$$

$$46 = 4 \text{ tens and } 6 \text{ ones}$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

### Doubles

When a number is doubled it is added to itself, so both addends are the same.

$$\text{The double of 4 is 8. } 4 + 4 = 8$$

$$\text{The double of 6 is 12. } 6 + 6 = 12$$

### Doubles Plus One

Use if one addend is one more than the other addend. Double the smaller addend and add 1 to the sum.

$$4 + 5 = 4 + 4 + 1 = 8 + 1 = 9$$

$$10 + 11 = 10 + 10 + 1 = 20 + 1 = 21$$

### Making Ten

Use when the larger addend is close to a multiple of 10.

Add to make a multiple of 10, then subtract the same number from the smaller addend.

$$9 + 4 = (9 + 1) + (4 - 1) = 10 + 3 = 13$$

$$38 + 6 = (38 + 2) + (6 - 2) = 40 + 4 = 44$$

### Landmark Numbers

Adjust both addends to make an equivalent problem that is easier to solve.

$$47 + 36 = (47 + 3) + (36 - 3)$$

$$= 50 + 33$$

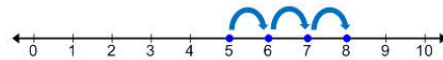
$$= 83$$

### Counting On

**Count on verbally.** Start with a known number and count on from that number. For example, if six pencils are added to a box of 12 pencils, it is not necessary to count the entire box of pencils. To find the total number of pencils say "twelve" and follow with "thirteen, fourteen, fifteen, sixteen, seventeen, eighteen." The total number of pencils in the box is 18.

**Use a number line to count on.**

$$5 + 3 = 8$$



### Partial Sums/Decomposing

Calculate sums by decomposing both addends and adding like place values. Use expanded notation to decompose each addend. Add like place values.

#### Horizontal Form

$$245 + 637 =$$

$$200 + 40 + 5$$

$$+ 600 + 30 + 7$$

$$800 + 70 + 12 = 882$$

#### Vertical Form

$$\begin{array}{r} 245 \\ + 637 \\ \hline 800 \\ 70 \\ + 12 \\ \hline 882 \end{array}$$

### Partial-Partial Sums/Decomposing

Calculate sums by decomposing only one of the addends.

$$326 + 418 =$$

$$326 + 400 = 726$$

$$726 + 10 = 736$$

$$736 + 8 = 744$$

### Rounding and Adjusting

Round each addend to a power of 10 and adjust the sum.

$$72 + 96 =$$

$$70 + 100 = 170$$

$$170 + 2 = 172$$

$$172 - 4 = 168$$

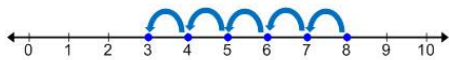
## Strategies for Subtraction You Might Encounter

### Counting Back

**Count back verbally.** Start with a known number and count back from that number. For example if a bag of marbles is known to contain 35 marbles and 6 marbles are removed, it is not necessary to count all the remaining marbles. Find how many marbles are left in the bag by counting back. Start with thirty-five following with, thirty-four, thirty-three, thirty-two, thirty-one, thirty, twenty-nine. There are 29 marbles left in the bag.

**Use a number line to count back.**

$$8 - 5 = 3$$



### Adding Up to Subtract

Add to the smaller number to reach a landmark number. Continue adding up to reach the larger number.

$$674 - 248 = \quad \$20.00 - \$14.39 =$$

$$\begin{aligned} 248 + 2 &= 250 & \$14.39 + \$0.01 &= \$14.40 \\ 250 + 400 &= 650 & \$14.40 + \$0.60 &= \$15.00 \\ 650 + 24 &= 674 & \$15.00 + \$5.00 &= \$20.00 \\ 2 + 400 + 24 &= 426 & \$5.00 + \$0.60 + \$0.01 &= \$5.61 \end{aligned}$$

### Landmark Numbers

Adjust both numbers to make an equivalent problem that is easier to solve by adding the same number to or subtracting the same number from both numbers.

$$67 - 32 =$$

$$\begin{aligned} 67 - 32 &= (67 - 2) - (32 - 2) \\ &= 65 - 30 \\ &= 35 \end{aligned}$$

### Rounding

Round one of the numbers to a power of 10 and adjust the difference.

$$58 - 39 =$$

$$58 - 40 = 18$$

$$18 + 1 = 19$$

### Partial Differences

Calculate differences by decomposing addends and subtracting like place values. Use expanded notation to represent each number. Subtract like place values. (Sometimes negative place values result). Add the partial differences.

#### Horizontal Form      Vertical Form

$$658 - 235$$

$$\begin{aligned} &600 + 50 + 8 \\ - &200 + 30 + 5 \\ \hline &400 + 20 + 3 = 423 \end{aligned}$$

$$\begin{array}{r} 658 \\ - 235 \\ \hline 400 \\ \phantom{0}20 \\ + \phantom{00}3 \\ \hline 423 \end{array}$$

### Negative Place Values

$$\begin{array}{r} 748 \\ - 492 \\ \hline 300 \\ - 50 \\ + \phantom{00}6 \\ \hline 256 \end{array}$$

### Partial-Partial Differences

Calculate differences by decomposing only the number being subtracted.

$$653 - 268 =$$

$$\begin{aligned} 653 - 200 &= 453 \\ 453 - 60 &= 393 \\ 393 - 8 &= 385 \end{aligned}$$

# Computation Strategies

## Strategies for Multiplication You Might Encounter

### Zero Rule

Any number multiplied by 0 equals 0.  
 $5 \times 0 = 0$

### Strategy for Ones

Any number multiplied by 1 equals that number.  
 $7 \times 1 = 7$

### Strategy for Twos (Doubling)

Any number multiplied by 2 is the same as doubling the number.  
 The double of 6 is 12:  $6 \times 2 = 12$

### Strategy for Fours (Doubling)

Any number multiplied by 4 equals the number doubled and then doubled again.  
 $6 \times 4 = (6 \times 2) \times 2 = 12 \times 2 = 24$

### Strategy for Fives

Any number multiplied by 5 equals half of the product of the number and 10.

$$8 \times 5 = \frac{1}{2}(8 \times 10) = \frac{1}{2}(80) = 40$$

### Strategy for Sixes (Doubling)

Any number multiplied by 6 equals the double of the product of that number and 3.

$$5 \times 6 = (5 \times 3) \times 2 = 15 \times 2 = 30$$

### Strategy for Nines

For all multiples of nine, the sum of the digits equals nine and the tens digit is one less than the multiplier.

$$3 \times 9 = 27$$

$$2 + 7 = 9$$

2 is one less than 3

### Multiplying On

Use a number line to multiply.

$$4 \times 3 = 12$$

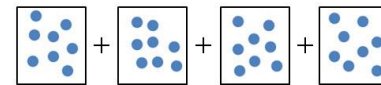


### Multiplication as Repeated Addition

**Skip count** to multiply as repeated addition.  $4 \times 8 = 32$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

**Find the sum** to multiply as repeated addition.  $4 \times 8 = 32$



$$8 + 8 + 8 + 8 = 32$$

### Partial Products

Decompose the factor or factors and multiply by place value.

Left to right

$$\begin{array}{r} 523 \\ \times 7 \\ \hline 3500 \quad (7 \times 500) \\ 140 \quad (7 \times 20) \\ + 21 \quad (7 \times 3) \\ \hline 3661 \end{array}$$

Right to left

$$\begin{array}{r} 523 \\ \times 7 \\ \hline 21 \quad (7 \times 3) \\ 140 \quad (7 \times 20) \\ + 3500 \quad (7 \times 500) \\ \hline 3661 \end{array}$$

Double digit

$$\begin{array}{r} 46 \\ \times 28 \\ \hline 48 \quad (8 \times 6) \\ 320 \quad (8 \times 40) \\ 120 \quad (20 \times 6) \\ + 800 \quad (20 \times 40) \\ \hline 1288 \end{array}$$

Decompose only one factor

$$\begin{array}{l} 15 \times 12 = \\ 10 \times 12 = 120 \\ 5 \times 12 = 60 \\ 120 + 60 = 180 \end{array}$$

## Splitting Factors

Split a larger factor into smaller number facts that are easier to multiply. Then add the partial products.

$$\begin{aligned} 16 \times 4 &= (8 \times 4) + (8 \times 4) \\ &= 32 + 32 \\ &= 64 \end{aligned}$$

## Array



Draw an array to model multiplication. Use rows and columns to represent the factors

$$4 \times 3 = 12$$

## Area Model

Use an area model to represent partial products. Then find the sum of the partial products.

$$56 \times 62 =$$

	60	2
50	$50 \times 60 = 3000$	$50 \times 2 = 100$
6	$6 \times 60 = 360$	$6 \times 2 = 12$

$$56 \times 62 = 3000 + 100 + 360 + 12 = 3472$$

$$7 \times 385 =$$

	300	+	80	+	5
7	$7 \times 300 = 2100$		$7 \times 80 = 560$		$7 \times 5 = 35$

$$7 \times 350 = 2100 + 560 + 35 = 2695$$

## Landmark Numbers

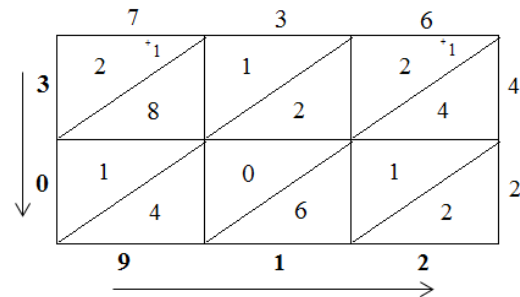
Decompose one factor to a landmark number that is easier to multiply. Use the Distributive Property to find the product.

$$\begin{aligned} 56 \times 7 &= (50 + 6) \times 7 \\ &= (50 \times 7) + (6 \times 7) \\ &= 350 + 42 \\ &= 392 \end{aligned}$$

## Lattice Method

1. Construct a lattice to fit the numbers being multiplied.
2. Multiply intersecting rows and columns. Enter the tens digit above the diagonal and the ones digit below the diagonal.
3. Add the diagonal paths from right to left. Carry to the next diagonal column if necessary.
4. The digits from the top left and across the bottom are the final product.

$$736 \times 42$$



$$736 \times 42 = 30,912$$

## Rounding and Adjusting

Round one factor to a multiple of ten and adjust the product.

### Round up and adjust

$$\begin{aligned} 67 \times 8 &= (70 \times 8) - (3 \times 8) \\ &= 560 - 24 \\ &= 536 \end{aligned}$$

### Round down and adjust

$$\begin{aligned} 42 \times 6 &= (40 \times 6) + (2 \times 6) \\ &= 240 + 12 \\ &= 252 \end{aligned}$$

## Strategies for Division You Might Encounter

### Repeated Subtraction

Repeatedly subtract the divisor from the dividend until there are no whole divisors remaining. Count the number of times you subtracted

$$28 \div 7 = 4 \qquad 35 \div 8 = 4 \text{ R}3$$

$$28 - 7 = 21 \qquad \begin{array}{r} 35 \\ - 8 \\ \hline 27 \end{array}$$

$$21 - 7 = 14 \qquad \begin{array}{r} 27 \\ - 8 \\ \hline 19 \end{array}$$

$$14 - 7 = 7 \qquad \begin{array}{r} 19 \\ - 8 \\ \hline 11 \end{array}$$

$$7 - 7 = 0 \qquad \begin{array}{r} 11 \\ - 8 \\ \hline 3 \end{array}$$

### Array (for small dividends)

Use counters to represent the dividend. Arrange the counters in rows to model the division.

$$21 \div 3 = 7$$



### Modeling (for large dividends)

Sketch empty figures to represent the divisor. Add the same number to each group until the sum of the items equals the dividend.

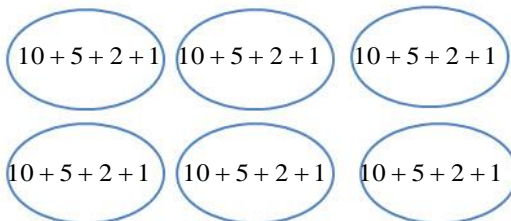
$$108 \div 6 = 18$$

$$6 \times 10 = 60$$

$$60 + (6 \times 5) = 90$$

$$90 + (6 \times 2) = 102$$

$$102 + (6 \times 1) = 108$$



### Partial Quotients (Also known as Big Seven Division and the Ladder Method)

Use known facts and multiples of 10 to divide large dividends. Find the sum of the partial quotients.

$$1124 \div 6 = 187 \text{ R}2$$

$$\begin{array}{r} 6 \overline{) 1124} \\ - 600 \quad 100 \times 6 \\ \hline 524 \\ - 300 \quad 50 \times 6 \\ \hline 224 \\ - 180 \quad 30 \times 6 \\ \hline 44 \\ - 42 \quad 7 \times 6 \\ \hline 2 \quad 187 \end{array}$$

### Partition the Dividend

Break the dividend into multiples of the divisor.

$$6 \overline{) 372}$$

$$\begin{array}{r} 60 + 2 = 62 \\ 6 \overline{) 360 + 12} \end{array}$$

$$158 \div 12$$

$$158 \div 12 = (120 + 38) \div 12$$

$$120 \div 12 = 10$$

$$38 \div 12 = \underline{3} \text{ R}2$$

$$= 13 \text{ R}2$$

### Area Model

Decompose the dividend into numbers that are divisible by the divisor. Use an area model to represent the partial quotients. Add the partial quotients.

$$3210 \div 15 = 214$$

	200	10	4
15	3000	150	60

$$200 + 10 + 4 = 214$$