

FIGHT "NEW" MATH FRUSTRATION

with this Mathematical Modeling Cheat Sheet

Building Strong Critical Thinking Skills for an Ever-Changing World

In our day, math education focused on rules, like $5 \times 5 = 25$, that never change.

Today, we know that our world is constantly changing. And math education teaches kids to look forward—to learn not just static rules, but flexible thinking and models they can use, change and improve in order to solve real-world problems.

In the future, our children will use spreadsheets, phones and new technologies we haven't even imagined yet, when they need to do calculations. They won't use stone tablets, an abacus, or a slide rule. Even calculators are becoming obsolete. (When was the last time you used an actual calculator, instead of your phone?)

Our children's careers will look very different in the future, as well.

"65% of children entering primary school today will ultimately end up working in completely new job types that don't yet exist," according to the World Economic Forum.



Today's math education is designed to help students think creatively, flexibly and logically, and to grow into confident and successful innovators, problem-solvers, and leaders.

We'll Help Break it All Down Through New Math Models

Mathematical modeling is introduced to younger students today because it is a **powerful tool for visualizing problems** and it provides guidance on how to think through and solve them!

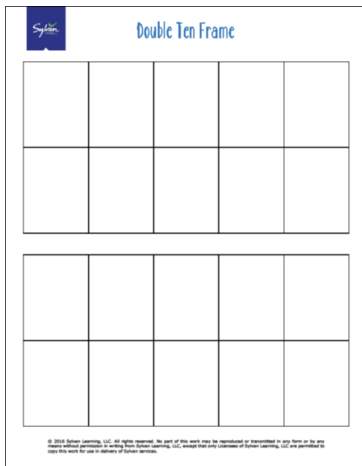
If you can build a model for a problem, you can build a machine (or design an app, or even start a new company) to do the work!



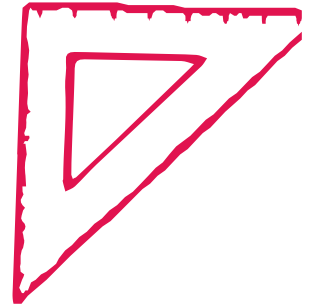
Brought to you by:

The mathematical modeling techniques your child will learn (and we break down for you in this guide) include:

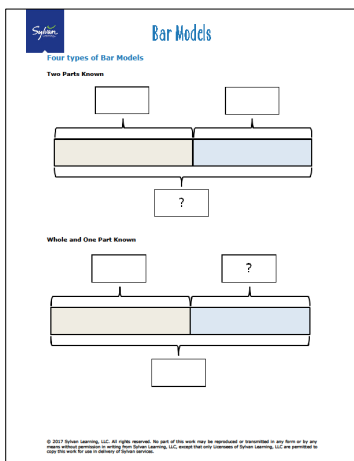
Double Ten Frame



The double ten frame is used with two-color counters to develop understanding of base ten and addition. Students use this model to visualize sums up to twenty using regrouping to make ten.



Bar Models

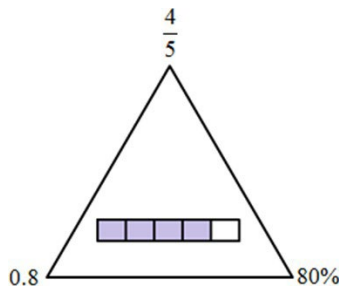


Bar models are used to help visualize word problems. The four different types of word problems that can use bar models are:

- Two parts are known and the whole is unknown
- The whole and one part are known
- Comparison: more than
- Comparison: fewer than



Portions Web



A portions web is a visual representation of one portion of a whole. The web shows four ways to represent a given portion in a triangular format:

- Picture or words
- Fraction
- Decimal
- Percent

The image helps students to remember that every number can be represented in multiple ways that have equivalent values.

THE MAKING TEN STRATEGY

Your child has to “show their work” but when you’re helping with homework, you may not know the “new” ways they are using. We break down one of the commonly used strategies: **Making Ten**.

Use this when the larger addend is close to a multiple of 10.

Add to make a multiple of 10, and then subtract the same number from the smaller addend.

$$9 + 4 = (9 + 1) + (4 - 1) = 10 + 3 = 13$$

$$38 + 6 = (38 + 2) + (6 - 2) = 40 + 4 = 44$$

That text was written for a teacher, not a student. Since this strategy is intended for students who are just learning their addition facts, it is important to break it down into something that does make sense to a young child. So let’s use another table tool, the Double Ten Frame and some double sided counters to show how we would explain this to a student.

Step 1:

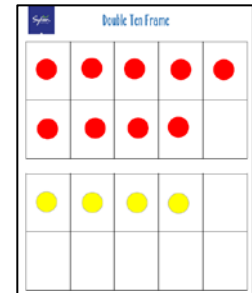
Make certain that the student understands that there are ten sections in each frame. Give students a handful of counters and have them place them in the frames as they count to 10 and then on to 20.

Next place 10 counters on the first frame and 2 counters on the second frame. The student should be able to count starting 10, 11, 12, without first counting from 1 to 10. Practice this skill before moving on to the next step.



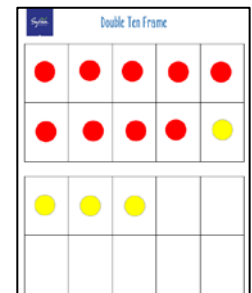
Step 2:

Set up the problem, $9 + 4$, by placing 9 red counters on the top frame and 4 yellow counters on the bottom frame. Explain to the student that there are 9 red counters and 4 yellow counters and we want to know how many counters there are altogether. Give the student time to process the problem and come up with a solution method. Praise any solution method that results in an answer of 13.



Step 3:

If the student did not use the making ten strategy present it as an alternative. Explain that we are going to learn a new strategy called Making Ten. This strategy tells us to move yellow counters from the bottom frame to fill in the top frame to complete ten making it easier to visualize the solution.



Step 4:

Students might be given a problem like: $9 + 4 = (9 + \underline{\quad}) + (4 - \underline{\quad})$ and asked to fill in the missing numbers. Using the double tens frame students can visualize that one yellow counter was “subtracted” from 4 counters and “added” to the 9 red counters.

$$9 + 4 = (9 + 1) + (4 - 1)$$

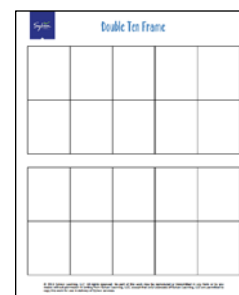
THE MAKING TEN STRATEGY

Here is an example showing $7 + 5$.

Step 1:

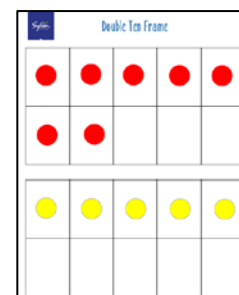
Make certain that the student understands that there are ten sections in each frame. Give students a handful of counters and have them place them in the frames as they count to 10 and then on to 20.

Next place 10 counters on the first frame and 2 counters on the second frame. The student should be able to count starting 10, 11, 12, without first counting from 1 to 10. Practice this skill before moving on to the next step.



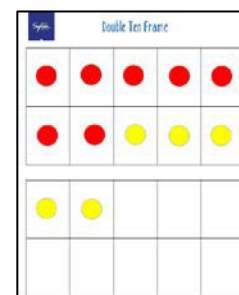
Step 2:

Set up the problem, $7 + 5$, by placing 7 red counters on the top frame and 5 yellow counters on the bottom frame. Explain to the student that there are 7 red counters and 5 yellow counters and we want to know how many counters there are altogether. Give the student time to process the problem and come up with a solution method. Praise any solution method that results in an answer of 12.



Step 3:

If the student did not use the making ten strategy present it as an alternative. Explain that we are going to learn a new strategy called Making Ten. This strategy tells us to move yellow counters from the bottom frame to fill in the top frame to complete ten making it easier to visualize the solution.



Step 4:

Students might be given a problem like: $7 + 5 = (7 + \underline{\quad}) + (5 - \underline{\quad})$ and asked to fill in the missing numbers. Using the double tens frame students can visualize that three yellow counter were "subtracted" from 5 counters and "added" to the 7 red counters.

$$7 + 5 = (7 + 3) + (5 - 3)$$

Once students understand this strategy they can apply it to larger sums, such as $38 + 6$, by adding the correct number to reach the next ten. In this problem, 2 would be added to 38 to reach the nearest ten, 40, and so 2 must also be subtracted from 6. Visualizing the movement of counters from one ten frame to the next makes the concept easier for students, and also for parents who did not learn this concept in school.

Bar Models are used to help visualize word problems. Use rectangular bars to model the relationship between known and unknown quantities and choose a strategy to solve the problem. There are four basic types of bar models:

- Two Parts Known
- Whole and One Part Known
- Comparison: More Than
- Comparison: Fewer Than

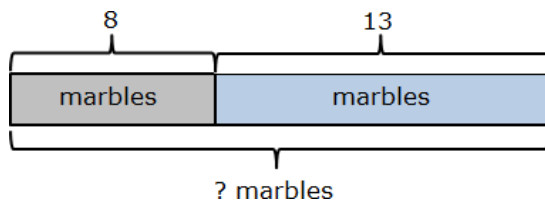
To use the bar model:

- Identify what is known and what is unknown
- Draw or select an appropriate model and label the known information
- Use the model to determine which operation to use to solve the problem

Example 1

Justin has 8 marbles and Charlotte has 13 marbles. How many marbles do they have altogether?

In this problem, the two parts are known. Select the Two Parts Known Bar Model. Use addition to find the unknown total.

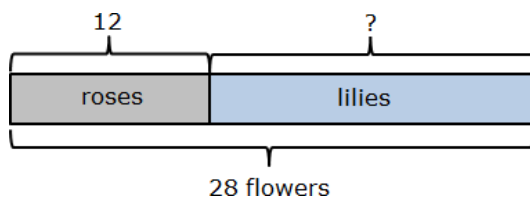


$8 + 13 = 21$ Justin and Charlotte have a total of 21 marbles.

Example 2

There are 28 flowers in the vase. Twelve of the flowers are roses. The rest are lilies. How many of the flowers are lilies?

In this problem the whole and one part are known. Select the Whole and One Part Known Bar Model. Use subtraction to find the missing part.

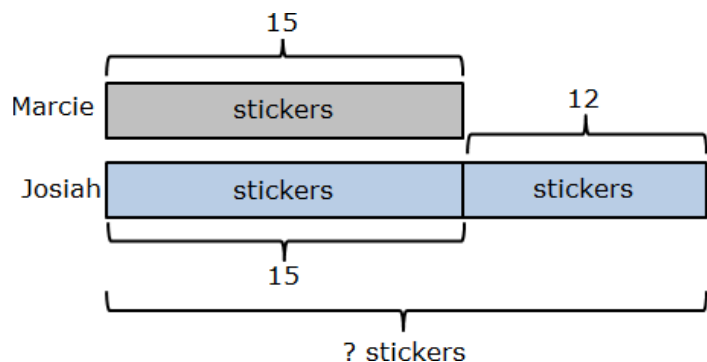


$28 - 12 = 16$ There are 16 lilies in the vase.

Example 3

Marcie has 15 stickers. Josiah has 12 stickers more than Marcie. How many stickers does Josiah have?

In this problem, “more than” is being used to compare two parts. Select the Comparison: More Than Bar Model. Use addition to find how many stickers Josiah has.

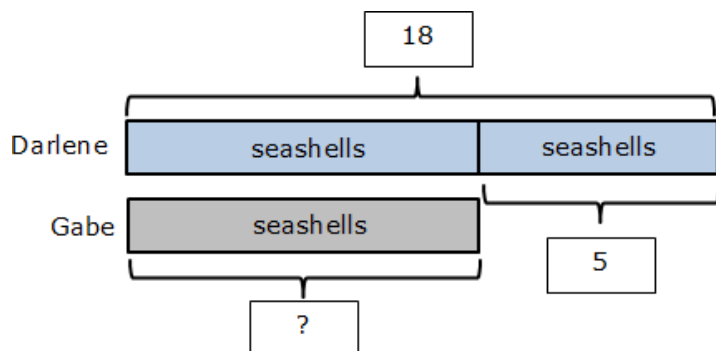


$15 + 12 = 27$ Josiah has 27 stickers.

Example 4

Darlene collected 18 seashells. Her brother Gabe collected 5 fewer seashells. How many seashells did Gabe collect?

In this problem, “fewer” is being used to compare two parts. Select the Comparison: Fewer Than Bar Model. Use subtraction to find how many seashells Gabe has.

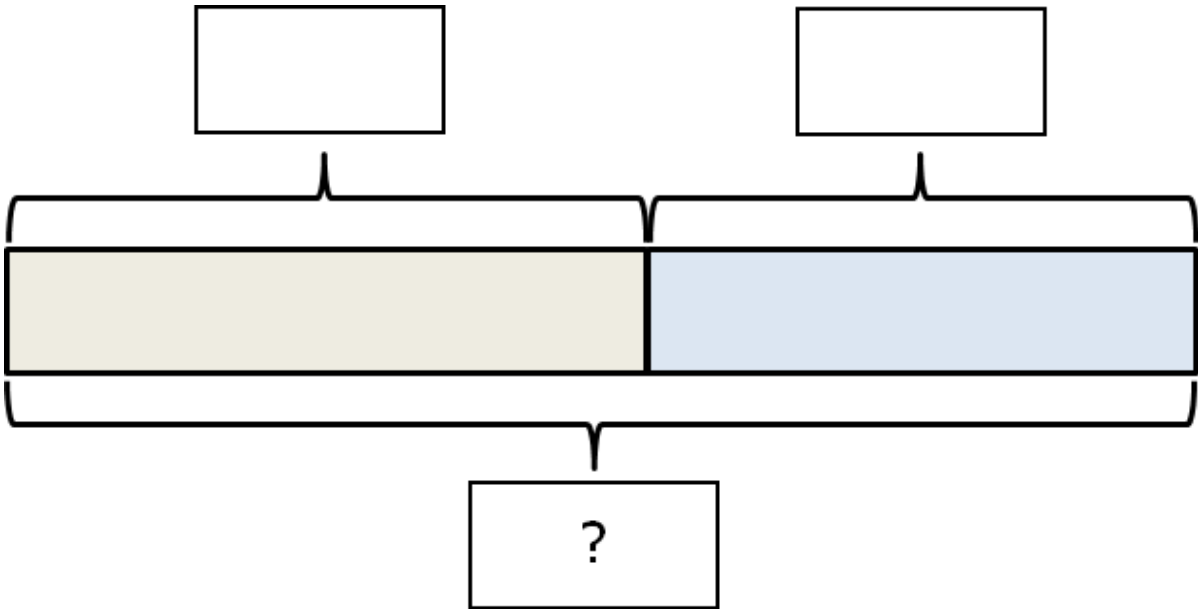


$18 - 5 = 13$ Gabe has 13 seashells.

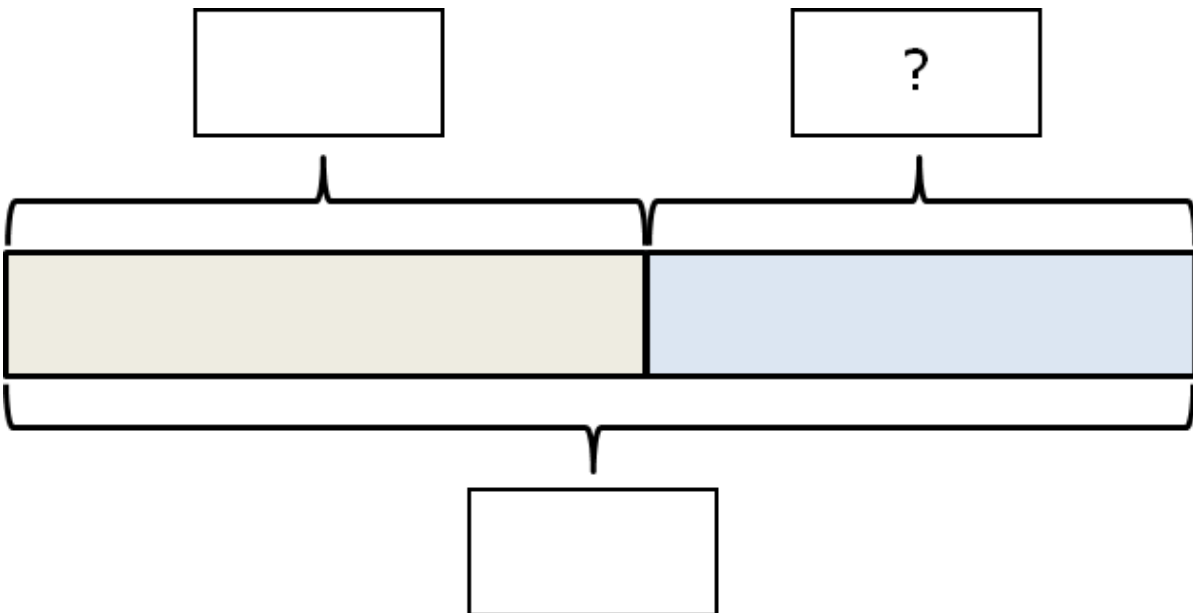
BAR MODELS

Four types of Bar Models

Two Parts Known

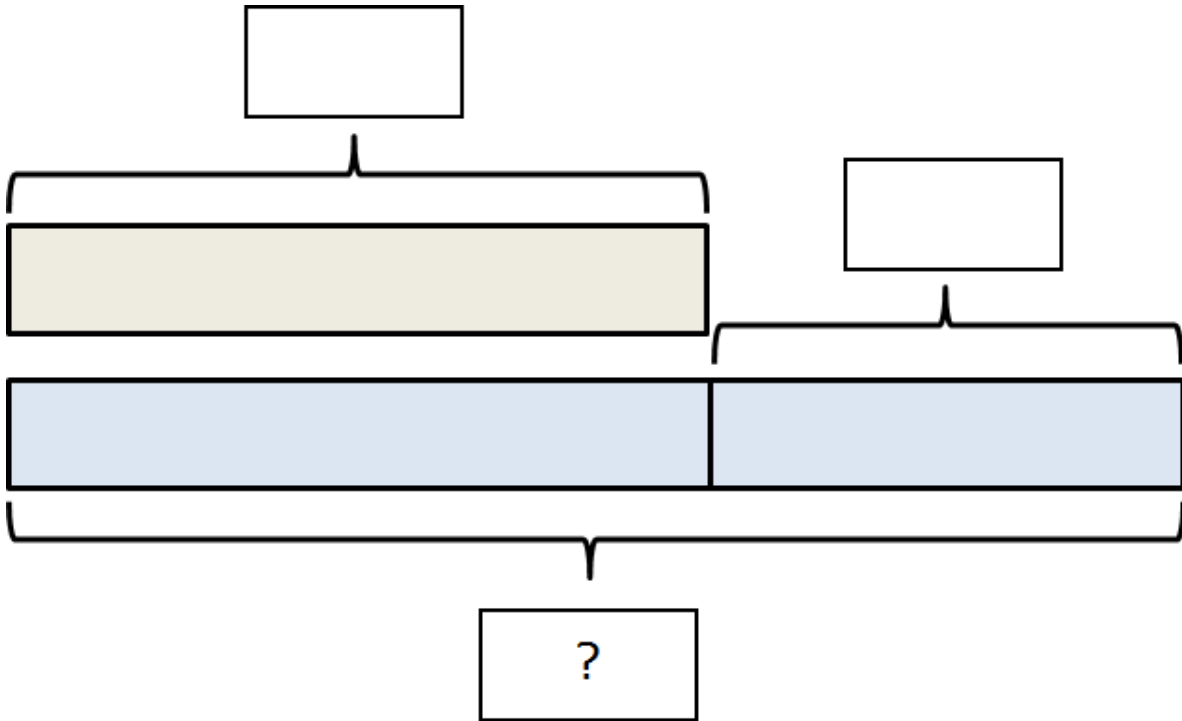


Whole and One Part Known

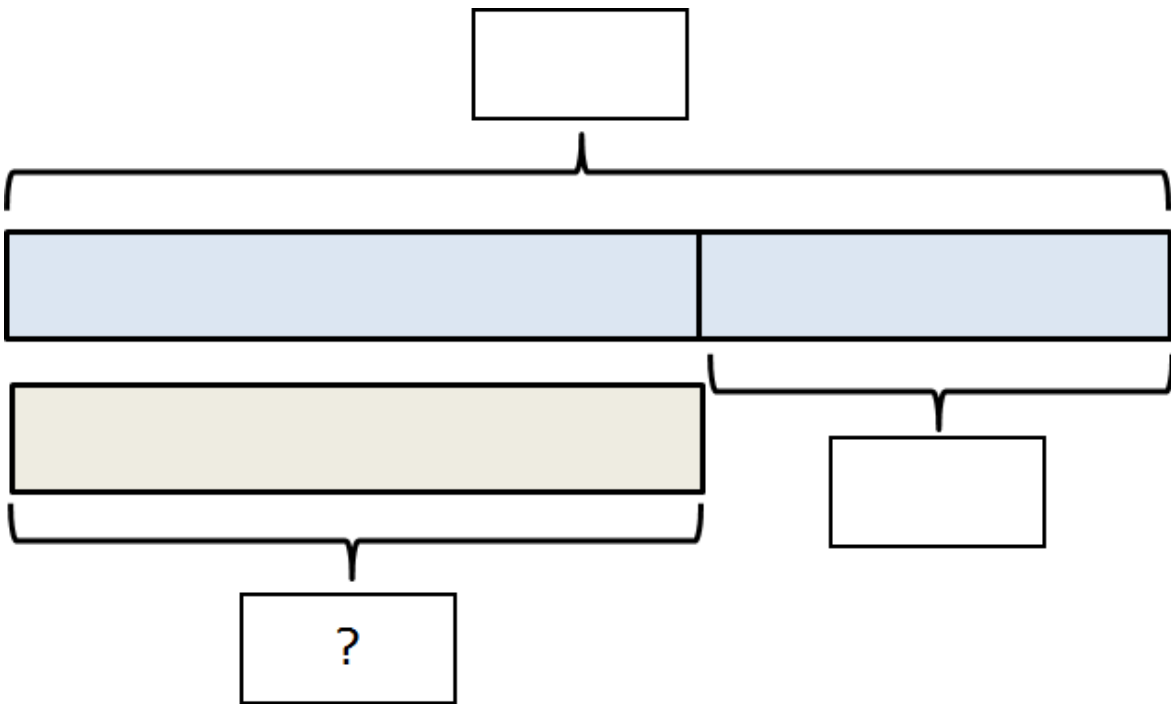


BAR MODELS

Comparison: More Than



Comparison: Fewer Than

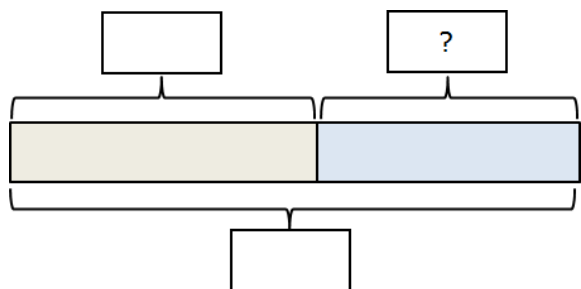
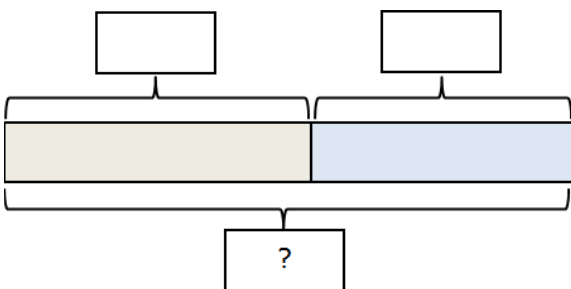
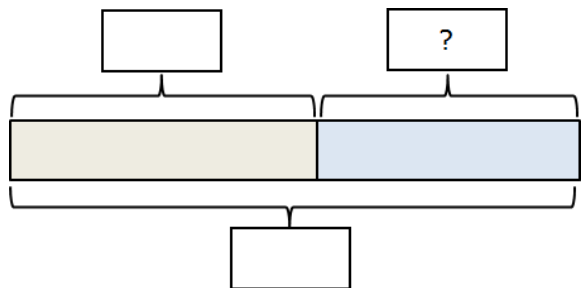
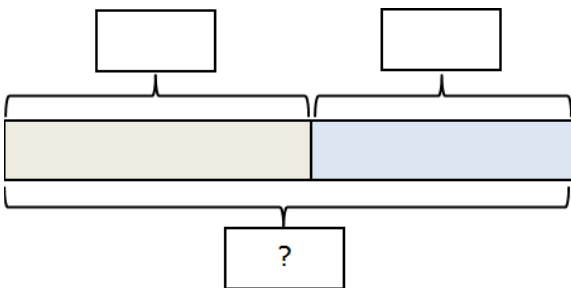
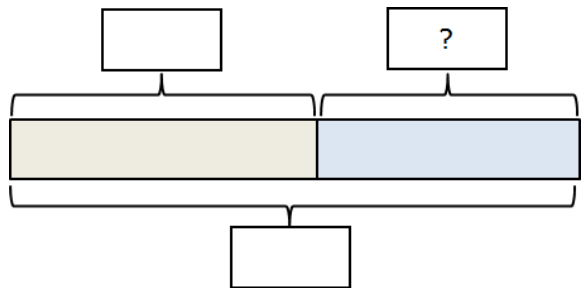
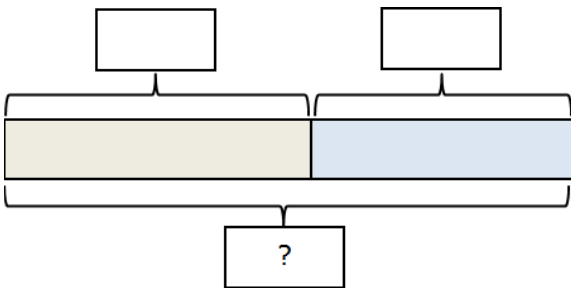
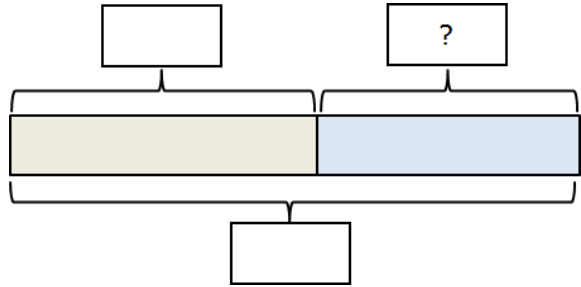
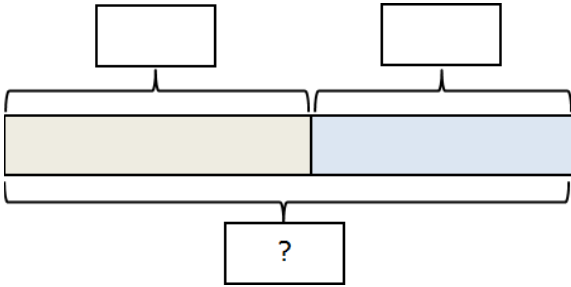


BAR MODELS

Use the following blank models to complete the practice problems.

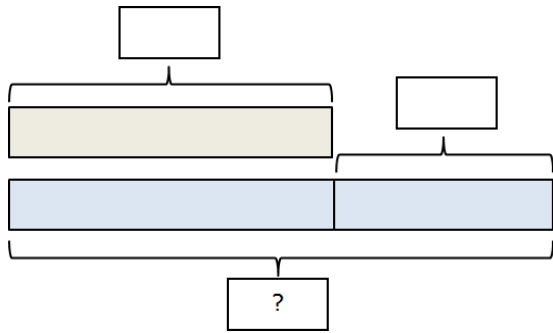
Two Parts Known

Whole and One Part Known

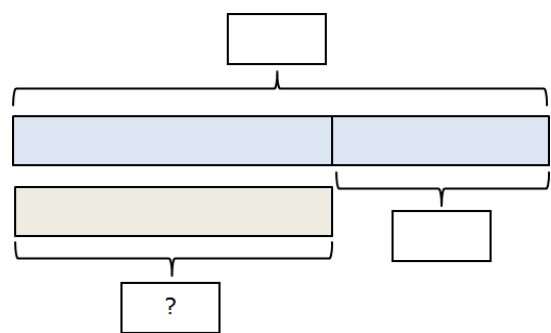
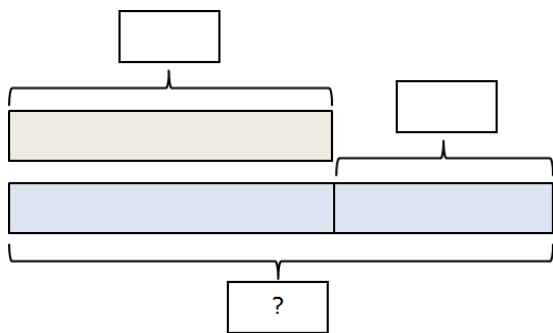
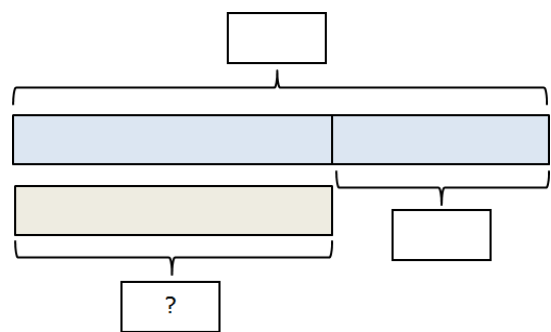
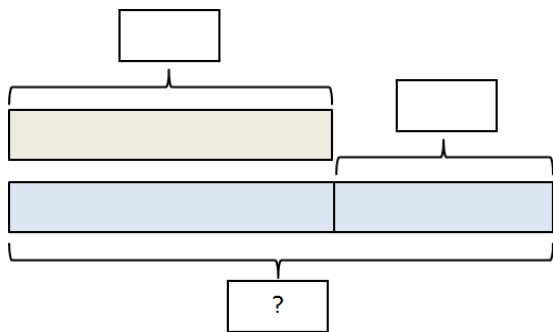
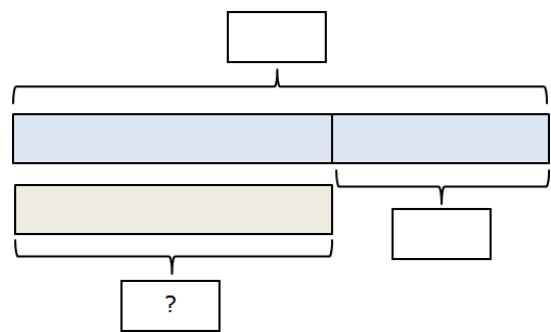
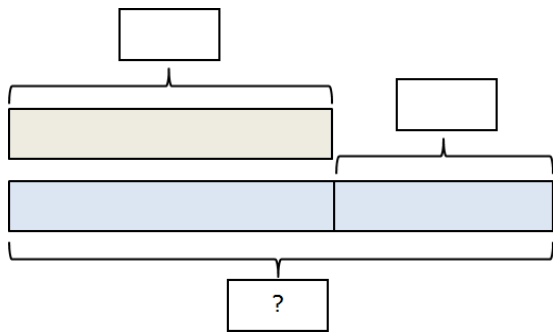
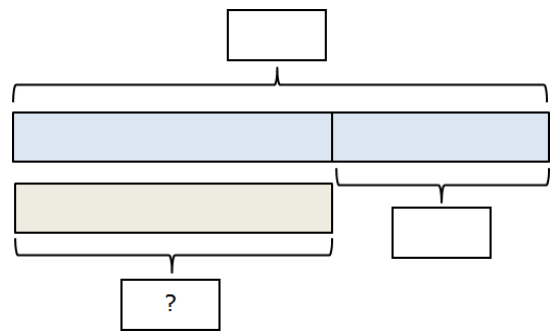


BAR MODELS

Comparison: More Than



Comparison: Fewer Than



Provide blank copies of bar models to complete the following practice problems.

Two Parts Known

1. Troy has 8 cards. Lily has 9 cards. How many cards do they have in all?
2. Monte had 24 stamps in his collection. His brother gave him 9 more stamps. How many stamps does Monte have now?
3. Michelle has 32 beads in her craft box. She bought another 24 beads. How many beads does Michelle have in all?
4. Frankie has saved \$68. His father gave him \$12 for cleaning the garage. How much money does Frankie have now?
5. There are 39 flavors of jelly beans at the candy store. The manager ordered an additional 15 flavors of jelly beans. How many flavors are in the store in all?

The Whole and One Part Known

1. Robbie had 18 balloons. He gave 7 of the balloons to his younger sister. How many balloons does Robbie have left?
2. Rafael had 32 baseball cards. He sold some of his cards to Mary. Now he has 25 cards left. How many cards did Rafael sell to Mary?
3. Mother made 96 cookies for the class party. The students ate 88 of the cookies. How many cookies were left?
4. Anna Marie had \$78 in her savings account. She spent \$42 on a new pair of shoes. How much money does Anna Marie have left in her account?
5. A total of 84 students competed in the marathon. Only 72 of the students completed the marathon. How many students did not complete the marathon?

Comparison: More Than and Fewer Than

1. There are 12 bluebirds sitting on the fence. There are 8 fewer sparrows than bluebirds on the fence. How many sparrows are sitting on the fence?
2. Franco has 50 toy cars. Bobbie has 38 more cars than Franco. How many cars does Bobbie have?
3. Lila has 35 pieces of bubble gum. Mike has 18 fewer pieces of gum than Lila. How many pieces of gum does Mike have?
4. Lawrence spent \$38 on a gift for his mother. Patty spent \$25 more than Lawrence on her gift. How much did Patty spend on her mother's gift?
5. There are 98 angel fish in the tank. There are 12 fewer goldfish than angel fish. How many goldfish are in the tank?

Students should identify the known and unknown in each problem, then choose a bar model and use it to solve the problem.

1. There are 24 cookies on the plate. Fifteen of the cookies are chocolate and the rest are sugar. How many sugar cookies are on the plate?

Which model should I use? _____

2. Francine has 38 pieces of colored paper. She bought 15 additional pieces at the school bookstore. How many pieces of colored paper does Francine have in all?

Which model should I use? _____

3. Paul bought 46 snack bars. Cary bought 35 more snack bars than Paul. How many snack bars did Cary buy?

Which model should I use? _____

4. Roseanne bought 75 pieces of rock candy. Justin bought 22 fewer pieces than Roseanne. How many pieces of rock candy did Justin buy?

Which model should I use? _____

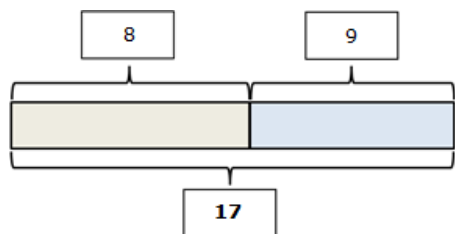
5. Maxine had 35 colored pencils in her art box. After she gave some of the pencils to her friend Jon, she had 19 pencils left. How many pencils did Maxine give to Jon?

Which model should I use? _____

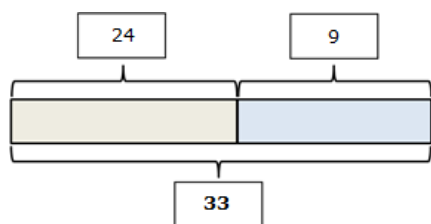
Practice Exercises Key: Answers are in bold print.

Two Parts Known

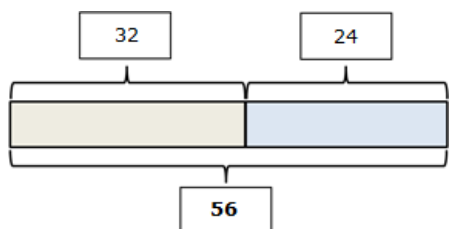
1. $8 + 9 = 17$



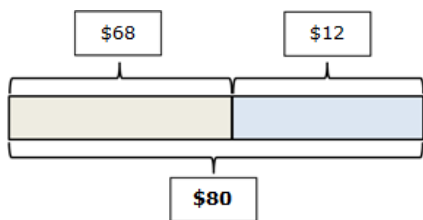
2. $24 + 9 = 33$



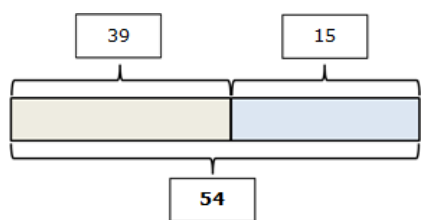
3. $32 + 24 = 56$



4. $\$68 + \$12 = \$80$

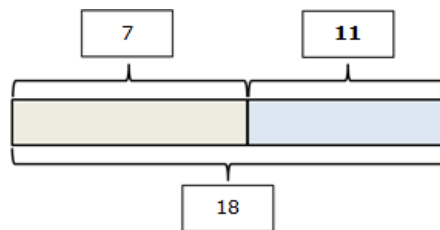


5. $39 + 15 = 54$

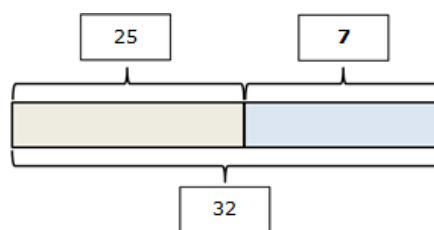


Whole and One Part Known

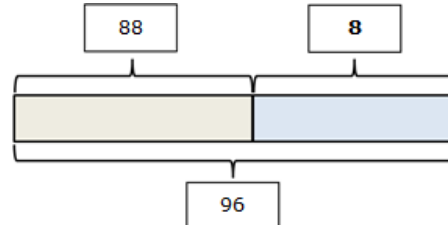
1. $18 - 7 = 11$



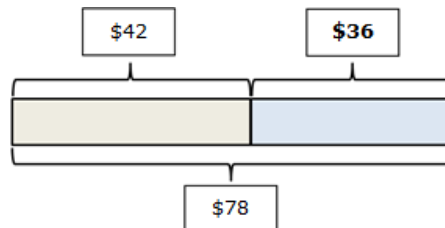
2. $32 - 25 = 7$



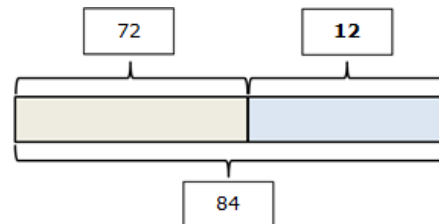
3. $96 - 88 = 8$



4. $\$78 - \$42 = \$36$



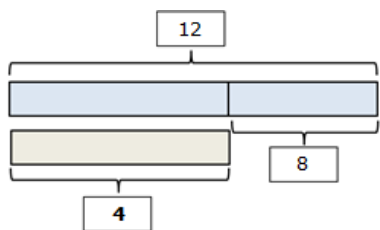
5. $84 - 72 = 12$



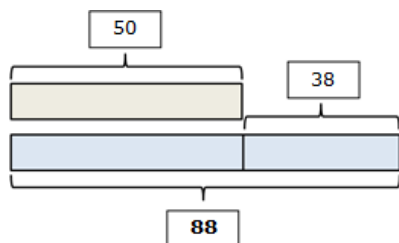
BAR MODELS

Comparison: More Than, Fewer Than

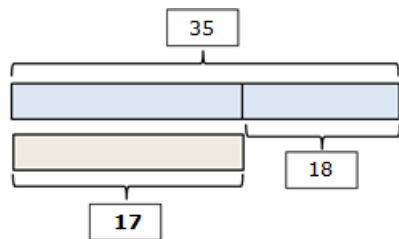
1. $12 - 8 = 4$



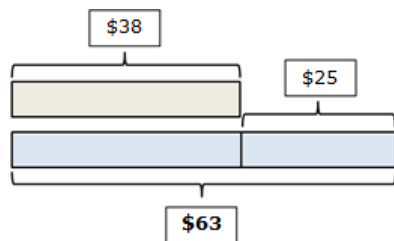
2. $50 + 38 = 88$



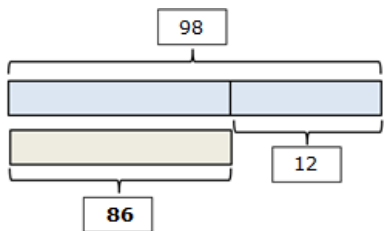
3. $35 - 18 = 17$



4. $\$38 + \$25 = \$63$

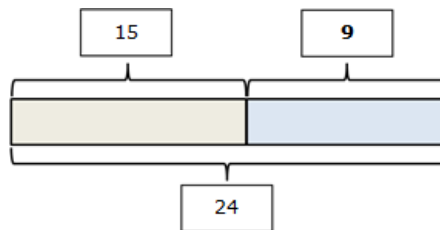


5. $98 - 12 = 86$

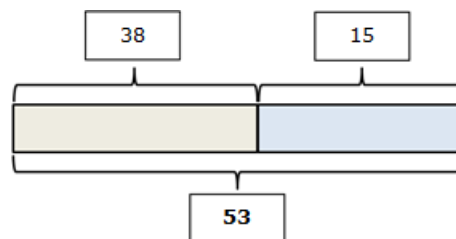


Choose the Model

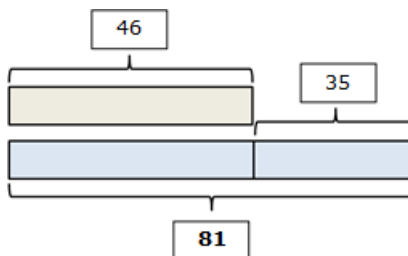
1. Whole and One Part Known $24 - 15 = 9$



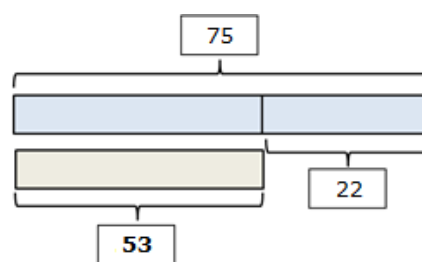
2. Two Parts Known $38 + 15 = 53$



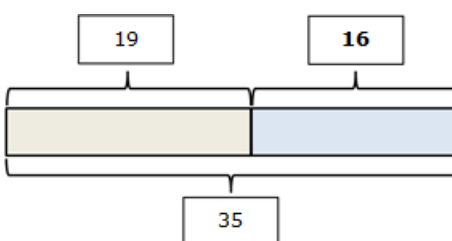
3. Comparison: More Than $46 + 35 = 81$



4. Comparison: Fewer Than $75 - 22 = 53$

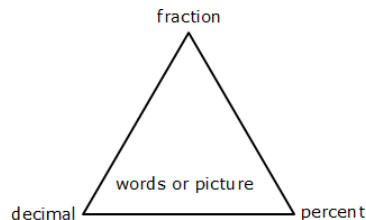
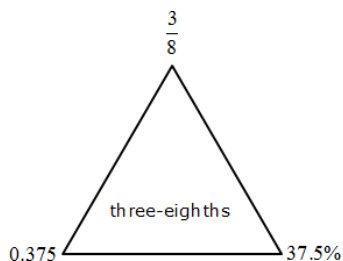


5. Whole and One Part Known $35 - 19 = 16$

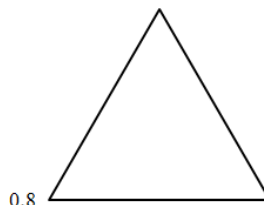


PORTIONS WEB

A **Portions Web** is a visual representation of one portion of a whole. The web includes four ways to represent a given portion in a triangular format: a picture or words, a fraction, a decimal, and a percent.



Example 1: Fill in the missing parts of the portions web.



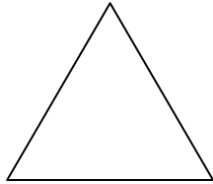
Fraction	Write the decimal as a fraction in simplest form. $0.8 = \frac{8}{10} = \frac{4}{5}$	
Percent	Move the decimal point two places to the right. $0.8 = 80\%$	
Words or Pictures	Use words or a picture to show the portion. 	

Example 2: Ronald planted 36 tomato plants in his vegetable garden. At the end of the season, Ronald noticed that only 27 of the plants had yielded tomatoes. What portion of the tomato plants that Ronald planted yielded tomatoes? Use a complete portions web to show your answer.

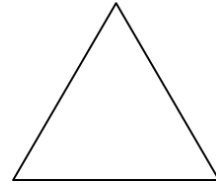
Fraction	Write the portion of the plants that yielded tomatoes as a fraction in simplest form. $\frac{\text{number of plants that yielded tomatoes}}{\text{number of plants Ronald planted}} = \frac{27}{36} = \frac{3}{4}$	
Decimal	Divide the numerator by the denominator. $3 \div 4 = 0.75$	
Percent	Move the decimal point two places to the right. $0.75 = 75\%$	
Words or a picture	Use words or a picture to show the portion. three-fourths	

Complete a portions web for each number.

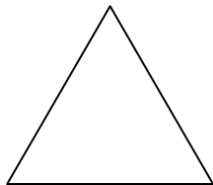
1. $\frac{5}{8}$



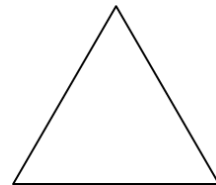
6. $\frac{13}{20}$



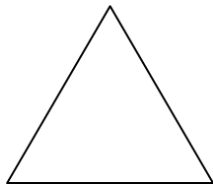
2. 82%



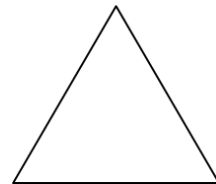
7. three-fifteenths



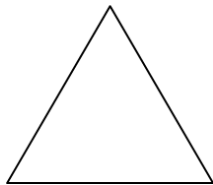
3. seven-tenths



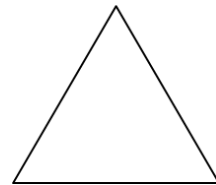
8. 98%



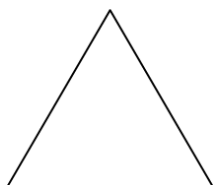
4. 0.68



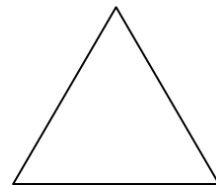
9. 0.125



5. 32.5%

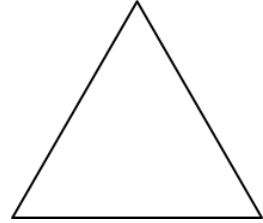


10. $\frac{28}{50}$

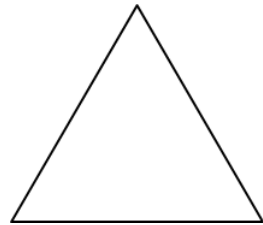


Solve each word problem. Use a complete portions web to show your answer.

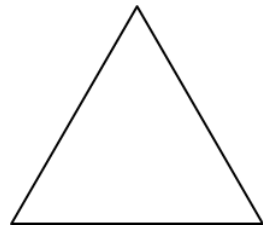
11. Margo is using colored pencils to complete an art project. When she opened the box of 48 colored pencils, she found that three of the pencils were broken. What portion of the box of colored pencils was broken?



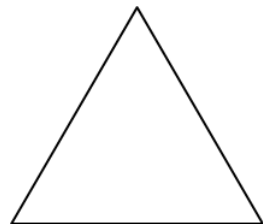
12. Janelle answered 88% of the question on her history test correctly. What portion of the test questions did Janelle answer correctly?



13. Fred gave $\frac{2}{5}$ of his trading cards to his sister Mary Lou. What portion of Fred's cards did he give to his sister?

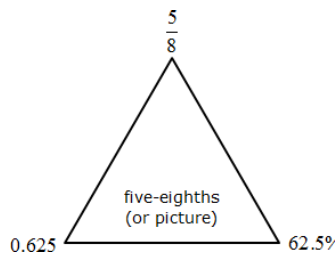


14. Frankie made 56 cookies for his birthday party. Fourteen of the cookies were peanut butter. What portion of the cookies that Frankie made were peanut butter?

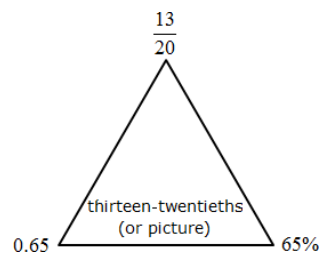


Answer Key

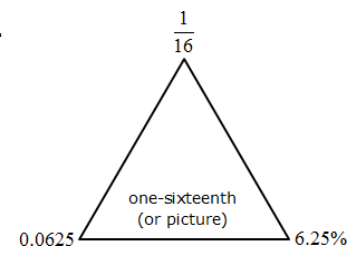
1.



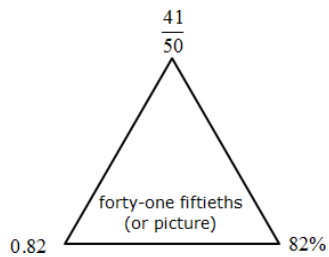
6.



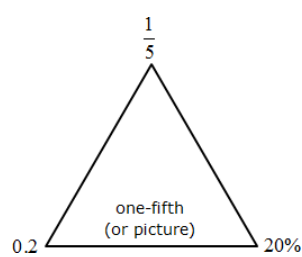
11.



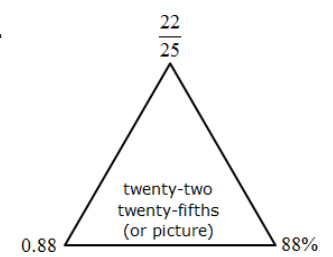
2.



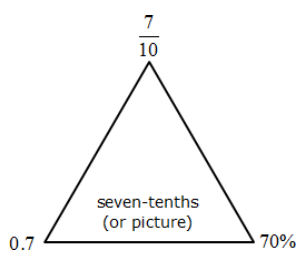
7.



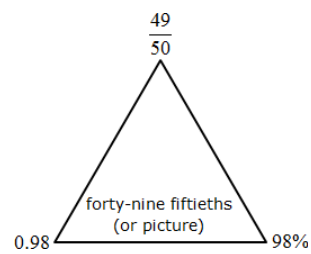
12.



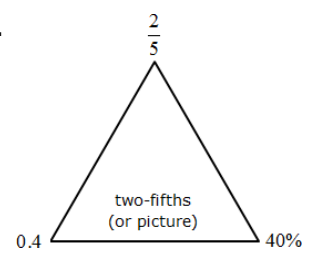
3.



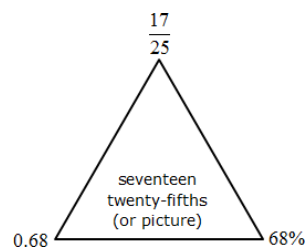
8.



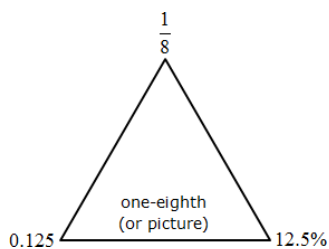
13.



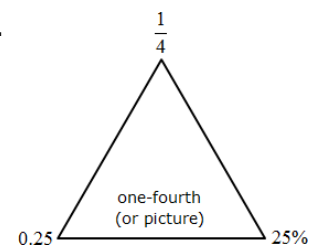
4.



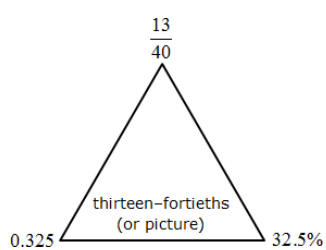
9.



14.



5.



10.

